Why Does an Electric Toy Car Move Uniformly?

Lea Valentina Lavrik, Vladimir Meir Shunyakov
lealavr@yahoo.com
Lifshitz Academic College, Jerusalem 2308, Israel
Tatiana Olevski, *learning center "Lingua. Com"*, Israel

ABSTRACT
We consider the problem of the stability of the uniform motion of a little electric toy car. Although the problem of stability of motion goes far beyond basic science, we were able to explain many aspects of this phenomenon, in the framework of high school physics. Our goal is to determine which forces influence the electric toy car; in particular, what are the sources of the braking force. We show that the stabilization of the car's motion is a manifestation of electromagnetic induction. In this work, we discuss the problem of the difference between uniform motion of the machine with the working engine and the uniform motion of bodies by the inertia as well.

INTRODUCTION

A. Einstein and L. Infeld had formulated the remarkable metaphor about scientific cognition in their book "The Evolution of Physics"[1].

"Physical concepts are free creations of the human mind, and are not, however they may seem, uniquely determined by the external world. In our endeavor to understand reality we are somewhat like a man trying to understand the mechanism of a closed watch. He sees the face and the moving hands, even hears its ticking, but he has no way to open the case. If he is ingenious he may form some picture of a mechanism which could be responsible for all of the things he observes, but he may never be quite sure his picture is the only one which could explain his observations…"

Our goal is to realize the Einstein-Infeld metaphor in practice, but instead of the watch, we will consider a little electric toy car. The main goal is to solve the problem of why does the electric car move uniformly.

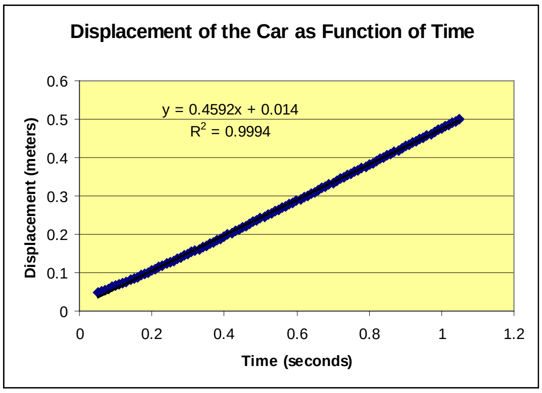
This problem is a practical one, because in physics lessons, we need to demonstrate uniform motion. The electric toy car is a most effective and cheap device for this demonstration. The uniform motion of the electrical toy car provides a repeatable, reliable and established result. Moreover, it is possible also to carry out many other demonstrations and lab assignments studying mechanics and electromagnetism by using electric toy cars.

In this paper, we consider in detail three different aspects of the problem of stability of uniform motion of the electric car:

1) The mathematical formulation of the principles of sustainable uniform motion.
2) The specific analysis of the forces acting on a moving car: in particular, what is the physical nature of the retarding force which depends on the speed?
3) The principle of inertia. What is the difference between the uniform, inertial motion of a body and the uniform motion of the electric car?

II. STABILITY CONDITIONS OF UNIFORM MOTION

The uniform and rectilinear motion of an electric car is an example of an established and repeatable proven result. The car's velocity may be different in different conditions: it depends on external loads and quality of the batteries. At the same time, the velocity does not change over the course of the motion.



Moreover, the car's uniform motion is stable if the resistive force depends on the car velocity. The car moves uniformly, if the forces, acting on it, balance each other. If the velocity is increasing, the magnitude of the resisting force is increasing, too. However, its sign is negative. It means that the sign of the resulting force is negative, too. If the resulting force is negative, it is pushing the car backwards. If the velocity is decreasing, then the resisting force is decreasing, but the resulting force is positive. The resulting force is pushing the car forward. By this way the uniform motion of the car is supported.

The mathematical aspects of this problem have been discussed by V.G. Boltyanski [2]. We should note that in the work [2], the physical nature of the pulling force and the resistance force is not discussed. In this paper, we focus principally on the physical nature of the forces that lead to the stabilization of uniform motion.

III. FORCES THAT INFLUENCE THE CAR'S MOTION

We know from our everyday experience that the rotating wheels of a car touch the road and push the road backward.

$$I\dot{\omega} = \tau_{motor} - \tau_{axis} - F_{stat}R \qquad (1)$$

Equation (1) describes the rotation of the wheel. Its moment of inertia is designated I, and the engine torque acting on the wheel is indicated by τ_{motor}. We suppose that the engine torque does not change and does not depend on velocity. The moment of the static friction force of the wheels on the road is indicated by F_{stat}, and the torque of friction between the rotating shaft and housing is indicated by τ_{axis}. The interaction force between the wheels and the road is the static friction force, F_{stat}. The force of static friction relates to the type of reaction force. The reaction forces are not dissipative forces, and according to d 'Alembert's principle [4], they do not produce any work in a system.

The road pushes the car forward, according to Third Newton's Law, by the static friction force. The equation for translational motion is

$$M\dot{v} = F_{stat} - F_{ext} - F_{resist}(v) \qquad (2)$$

In equation (2), M is the mass of the cars, and F_{ext} is the force due to an external load. We do not know the physical nature of the resistance force F_{resist}, which depends on the speed. It may be, for example, the force of air resistance. The velocity of the wheel at the point where it touches the road is equal to zero, because at the contact point the wheel's rotating velocity balances the car's forward velocity. Balance between speeds of translational and rotational motions at the point of contact between the wheel and the road is described by the following equation

$$v - \omega R = 0 \qquad (3)$$

Equations (1), (2), (3) describe the mechanical motion of the car. Equations (1) and (2) represent Newton's Second Law for the wheel's rotation and for the car's forward motion. Equation (3) represents the constraint condition between rotation and forward motions, and it shows mutual dependence between these kinds of motion.

If the resistance force depends on velocity, then Equation (2) satisfies the stability condition of uniform motion, which was formulated in the previous section. Equation (2) implies that as a constant external force is directed against the forward movement, the constant speed of the car should decrease.

Let us check this dependence. We connect a rope to the back part of the car. The rope passes over a pulley. The weight represents the set of rings on the hanger, and it increases as the car moves. We change the weight by adding a new ring and then we repeat this procedure.

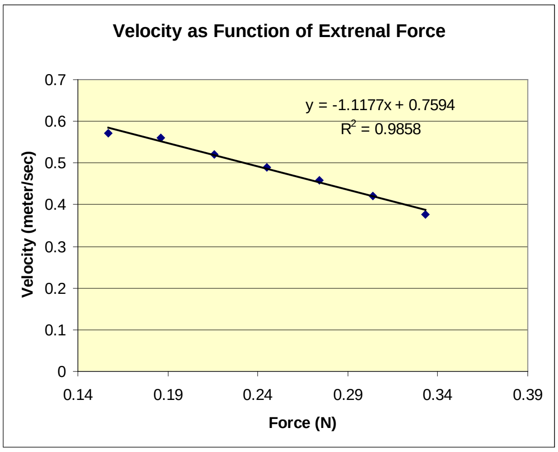


Figure 1: The speed of car depends on the external force, which pulls the car backward.

The main result of this experiment is the following: the magnitude of the velocity decreases linearly as the weight increases. From the plot in Fig. 1, we can estimate the value of a constant force that pulls the car forward. Strictly speaking, a linear relationship between the velocity and external load takes place only over a small range of forces. A further increase in weight leads to a transition to the regime of a sliding motion, and then to a full stop. The full stop comes when the load is from 0.4 - 0.5 N. Thus, from this experiment, we receive an order of magnitude of the force, which pulls the car forward. Let us consider the case where this force is equal to 0.4 N.

According to the Newton's First Law, any body moves uniformly if a resulting force influencing its motion is equal to zero. The resistance force, which pushes the car backward, may be only the force of air resistance. The air resistance force is the quadratic function of a body velocity [3]:

$$F_{air} = -\mu S \frac{\rho_{air} |v| \vec{v}}{2} \quad (4)$$

where μ ≈0.4 - the dimensionless factor that depends on the shape of the body.

The quantity S is the area of a frontal section of the body, which is perpendicular to the velocity of the moving body. A frontal cross section of the toy car is on the order of magnitude of about $S \approx 25\ cm^2 = 2.5 * 10^{-3} m^2$. The equation also includes the air density, $\rho_{air} = 1.3 kg/m^3$.

The air resistance force increases in magnitude as the speed of the car increases. At a certain critical velocity, a pulling force coincides to the resistance force, and motion becomes uniform. Let us verify whether air resistance is significant in the motion of the toy car.

In the third section, we estimated the order of magnitude for the force pulling a toy car. Assuming that the pulling force is constant and does not depend on the speed, we see that its magnitude is 0.4 N. We can find the speed at which both forces are balanced. We equate the pulling force = 0.4 N, and the air resistance force (7). The speed at which both forces are balanced is 28 m / sec. This value is much higher than the characteristic speed at which an electric toy car moves, at 0.2-1 m / sec. At such speeds, air resistance is much smaller than the pulling force, and therefore its influence can be neglected.

IV. HYPOTHESIS ABOUT THE PROPERTIES OF TORQUE CREATED IN ELECTRIC MOTORS

We can eliminate the angular velocity from the system of equations in order to obtain only one effective equation.

$$\left(M + \frac{I}{R^2} \right) \dot{v} = \frac{\tau_{motor} - \tau_{axis}}{R} - F_{ext} \quad (5)$$

The effective driving force is proportional to the torque which rotates the wheels. If the car moves uniformly, then the driving force and the torque are equal to zero. This effective equation corresponds to simple intuition that “the engine pulls the car”. In this case, we forget about the road's impact on the wheel, and we assume that the motor is the only source of the pulling force.

The total torque which acts on the wheel is smaller than the torque created by the motor. It overcomes the torque of sliding friction on the wheel's axis and on the transmission. An effective pulling force is associated to the total torque. The external force F_{ext} counteracts the movement. In spite of the fact that the force of static friction has been eliminated from the effective equation (10), it can be found by solving equations (1), (2) and (9).

$$F_{stat} = (1-\lambda)\frac{\tau_{motor} - \tau_{axis}}{R} + \lambda F_{ext}; \lambda = \frac{I}{MR^2 + I} \qquad (6)$$

Equation (6) is valid when the right side does not exceed the value of $\boldsymbol{\mu_{stat} * Mg}$. Using equation (6), we can solve a curious paradox. We face this paradox in the study of bodies moving on wheels. As mentioned above, the main source of external forces affecting the car's movement is the road surface. Nevertheless, at the same time, we are able to put the car in motion, to accelerate its movement, to slow down the motion, or to stop it. The question is: if all this is happening due to the impact of the road, how the road does “know" when it is necessary to stop the car, or when it does need to speed it up or slow it down?

Equation (6) gives the answer to that question. The answer is that the static friction force is the reaction force. It was determined through the moment force applied to the wheel and through the other outside forces. The moment of force can be controlled by sitting in the car, and thus to manage the magnitude and direction of the static friction force. Thus, if the machine moves with constant velocity, and the external force applied to it is zero, then the total torque applied to the wheel must also be zero.

This conclusion leads to a new paradox. If the torque generated by the motor is constant in magnitude, then uniform motion of the car is virtually impossible. Since in the original equations we did not find any forces that depend on speed, then it seems reasonable to formulate the following hypothesis: the torque applied to the wheel by the motor depends on the angular rotational speed, and decreases with its growth. For simplicity, assume that the torque decreases linearly with increasing angular velocity. Without loss of generality, we propose an expression for the torque, which decreases linearly with increasing angular velocity, as follows.

$$\tau_{motor} = \tau_0\left(1 - \frac{\omega}{\omega_0}\right) \qquad (7)$$

Here τ_0 may be interpreted as the maximal value of the torque which is produced by the motor. We can estimate it by measuring the minimum moment of external forces which stop the rotation of the running engine. The parameter ω_0 may be interpreted as the maximum angular velocity of the motor. It is practically impossible to achieve this angular velocity, because even in the absence of external loads, there is internal friction. The effective equation takes the form

$$\left(M + \frac{I}{R^2}\right)\dot{v} = \frac{\tau_0 - \tau_{frict}}{R} - \frac{\tau_0}{\omega_0 R^2}v - F_{ext} \qquad (8)$$

The static friction force (11) also contains a "viscous contribution" due to the dependence of the torque on the angular velocity

$$F_{stat} = (1-\lambda)\left(\frac{\tau_{motor} - \tau_{axis}}{R} - \frac{\tau_{motor}}{\omega_0 R^2}v\right) + \lambda F_{ext}; \qquad (9)$$

Thus, only the static friction force influences the motion of an electric car. Besides the static friction force, no other external force influences the motion of the car. Moreover, the static friction force represents the difference of two terms. The first of these can be interpreted as a 'pulling force', which pushes the machine forward. The second term is the "friction force" that depends on the speed. It remains an open question as to why the torque generated by the motor (6) depends on the angular velocity of the wheel.

V. DYNAMICS OF THE MOTOR

Schematically, the electric DC motor represents a conductive frame which can freely rotate in a magnetic field. When an electrical current flows along the frame, the magnetic field produces a torque which influences the frame rotation. The torque is proportional to the electric current, according to the Ampere's Law [7].
In order to obtain rotational motion, you need to change the polarity of the contacts on the inner rotating frame every half period. This may be achieved by using sliding contacts - brushes, which change polarity on the inner contacts of the rotating coil. In this case, the torque acting on the frame with the current keeps the same sign throughout the entire process of movement. We write the expression for the torque as

$$\tau_{motor} = I\Phi_{\max}\left|\sin(\theta)\right| \ . \qquad (10)$$

In the circuit containing the motor, there are two sources of electricity. The first source is a battery. The second source of electricity in the circuit is the engine itself, more specifically, the magnetic field of the rotating coil. The rotating coil creates an EMF by electromagnetic induction in the circuit. Its sign is opposite to the battery's EMF, and its magnitude is proportional to the angular velocity of the frame.
Ohm's law for this circuit is given by

$$I(R+r) = \mathcal{E}_{batt} - \mathcal{E}_{ind} = \mathcal{E}_{batt} - \omega\Phi_{\max}\left|\sin(\theta)\right| \qquad (11)$$

Here "R" and "r" denote the resistance of the coil and the internal resistance of the batteries, respectively.
Because the torque is proportional to the electrical current by Ampere’s Law (10), we obtain the dependence of the torque on the angular velocity of the rotation. We can write the expression for the torque, averaged over the various orientations of the frame, in an explicit form.

$$\tau = \tau_{\max} - \omega\frac{\tau_{\max}}{\omega_0} = \frac{2}{\pi}\frac{E_{batt}\Phi_{\max}}{R+r} - \omega\frac{\Phi^2_{\max}}{2(R+r)} \qquad (12)$$

Equation (12) shows that the torque created by the motor consists of two parts. One part is constant and it makes the wheel rotate. The second part depends on the angular velocity. It is negative and prevents rotation.
The static friction force can be decomposed into two parts. One of them pulls the car forward, while the other depends on the speed and pushes the car backward. This is the solution to the puzzle regarding the stability of uniform motion of the machine.

Otherwise, we can say that a stable and uniform motion of the car is essentially a manifestation of the law of electromagnetic induction.

VI. DISCUSSION

Formally, the uniform motion of an electric toy car satisfies the conditions for inertial motion. Nevertheless, our intuition and common sense do not agree with this formal conclusion. The objection that immediately comes to mind is: why does a car stop immediately upon turning off the engine? It turns out that inertial motion occurs only while the motor is running.

What is the difference between the "true movement of inertia" and the uniform motion of the car on a level road? Strictly speaking, the uniform motion of an electric car is only an average. The static frictional force and torque which act on the wheel from the road and from the engine are pulsing functions of time. A non-trivial feature of the system being studied is that the magnitude and direction of an external force, which is the static friction force, can be controlled from inside the car. And, although managing the external force from within the system is not such a rare phenomenon, the discussion of this phenomenon is not in the curriculum even in many university courses.

Moreover, in physics courses they usually assume that uniform motion is realized only under the condition where two or more external forces strictly counterbalance each other. But by using this assumption, we are not able to explain many of the usual phenomena occurring in the world around us. Simplifications are inevitable, and it is impossible without them.

In elementary physics courses, the motion of complex objects is seen as movement of a point mass acted upon by an external force. When a teacher explains the motion of a person walking along the road, he says that the static friction force acts on person's feet from the ground. However, the instructor must be prepared to respond to another completely obvious question: what force balances the static friction force when a person moves at a constant speed? This is the subject of this paper.

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